Cartesian Product

Let A and B be two non empty sets then cartesian product of A and B is denoted by $A \times B$ and is defined as $A \times B = \{ (x , y) / x \in A \text{ and } y \in B\}$

Binary Relation (Relation)

Any subset of $A \times B$ is called a binary relation from A to B. It is usually denoted by 'r'

Domain

The set of the first elements of the ordered pairs of a relation is called its domain.

Range

The set of second elements of the ordered pairs of a relation is called its Range.

Relation on a Set

If A is a non empty set then any subset of $A \times A$ is called a relation in A or relation on A.

Example:

Let $A = \{1, 2\}$, $B = \{3, 4\}$

$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$
Relation = $r = \{(1, 3), (2, 4)\}$
$D_r = \{1, 2\}$
$R_r = \{3, 4\}$

(Fedoral 2014, Bwp 2016)

Function

Let A and B be two non empty sets such that
i). f is a subset of $A \times B$
ii). Dom f = A
iii). First element of no two pairs of f are equal.
Then 'f' is said to be a function from A to B.
Function is usually written as
$f: A \to B$ (read as f is a function from A to B)

Into Function

A function $f: A \to B$ is said to be into function if Ran $f \subset B$ i.e Ran $f \neq B$.

(Lhr 2015 G II)

Onto Function (Surjective function)

A function $f: A \to B$ is said to be onto if Ran f = B

(OR)

A function $f: A \to B$ is said to be onto if every element of B is the image of some element of A.

(1-1) Function (Injective function)

A function $f: A \to B$ is said to be one–one if second elements of no two ordered pairs are equal.

(1-1) and onto Function (Bijective function)

A function 'f' from A onto B is said to be bijective function if second element of no two of its ordered pairs are the same.

(OR)

Both (1–1) and onto function is called bijective function.
Such a function is also called a (1–1) correspondence between A and B.

Set Builder Notation for a Function

When a function comprise infinite number of ordered pairs we write it in set builder notation. Consider the function.
$f = \{(1, 1), (2, 4), (3, 9), (4, 16), \ldots\}$
Domf = $\{1, 2, 3, 4, \ldots\}$, Ranf = $\{1, 4, 9, 16, \ldots\}$
In set builder notation function is
$f = \{(x, y) / y = x^2, x \in N\}$

Linear Function

The function $\{(x, y) / y = mx + c\}$ is called a linear function because its graph is a straight line.

Quadratic Function

The function $\{(x, y) / y = ax^2 + bx + c\}$ is called a quadratic function because it is defined by a quadratic equation in x, y.

Inverse of a Function

The inverse of a function f is denoted by f^{-1} and is obtained by interchanging the components of each ordered pair if the function is in tabular form. The inverse of a function may or may not be a function.
If $f = \{(1, a), (2, b)\}$ then $f^{-1} = \{(a, 1), (b, 2)\}$
If $f = \{(x, y) / y = mx + c\}$ then $f^{-1} = \{(x, y) / x = my + c\}$

Note:
1. Inverse of a line is a line clearly.
2. $y = x$ is a self inverse function.

Identity Function

The function $\{x, y) / y = x\}$ is called the identity function.